Deets and Hazard

Verification of the Synchronous Machine Circle Diagram

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VERIFICATION OF THE SYNCHRONOUS MACHINE CIRCLE DIAGRAM

BY

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THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

RALPH EMERSON DEETS and LEE HERBERT HAZARD

ENTITLED VERIFICATION OF THE SYNCHRONOUS MACHINE CIRCLE DIAGRAM

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF Bachelor of Science in Electrical Engineering

Morgan Brooks
Instructor in Charge.

APPROVED: Morgan Brooks

HEAD OF DEPARTMENT OF Electrical Engineering

INTRODUCTION.

The object of this thesis is to test experimentally the theory of the Polar Diagram as presented by Professor Morgan Brooks at the 24th Annual Convention of the American Instutute of Electrical Engineers at Niagaria Falls, in a paper entitled Interaction of Synchronous Machines.

For the purposes of this investigation, a series of tests have been made, and from the data taken, diagrams have been plotted, using the graphical methods of the Polar Diagram as a basis. By comparing these diagrams which show actual operating conditions, with those based upon theoretical considerations alone, the authors will endeavor to show how closely the theory of the Circle Diagram does represent the interaction of synchronous machines, and through what range of operation it may be depended upon to give reasonably exact results.

This paper will deal with the operation of synchronous machines as generator and motor, only, and in all cases the generator will be used as a reference machine. The problem will be treated from two different standpoints; first, the angular difference between the E. M. F. vectors of the two machines will be measured mechanically by means of differential gearing and these angles used in constructing diagrams; second, all data necessary for the construction of another set of diagrams will be derived from electrical measurements taken simultaneously with the data for the first set of diagrams. The second set of diagrams will be independent of the first, but will represent

exactly the same operating conditions, and are a second means of testing the theory.

DESCRIPTION OF APPARATUS.

For all of the tests which have been made two exactly similar synchronous converters have been used. These converters are four pole machines built by the General Electric Co. of Schenectady, so constructed that they may be used either on single phase, two phase, or three phase circuits. In every case the machines have been run single phase, one being used as an alternating current generator and the second as a synchronous motor. The machines are rated as follows: Direct Current - 68 Amperes, 110-160 Volts; Alternating Current - 60 cycles, 110 Volts; 1800 R. P. M., 75 K.W.

Plate I is a photograph of the differential gearing used to measure the angle between the E M. F. vertors of the generator and the motor while Plates II and III show the manner in which it is connected between the two machines. The machines are operated so that the shafts turn in opposite directions. The gear A' (Plate II) rotates with the shaft of machine A while the gear B' rotates with the shaft of machine B. If the machines are run at exactly the same speed, the frame and pointer as shown in Plate I remain stationary, i.e., do not revolve. Any difference in speed between the machines is shown by a slow rotation of the frame, the relative difference in speed between the two machines being communicated by means of the gears A and B through C to the frame which carries the pointer.

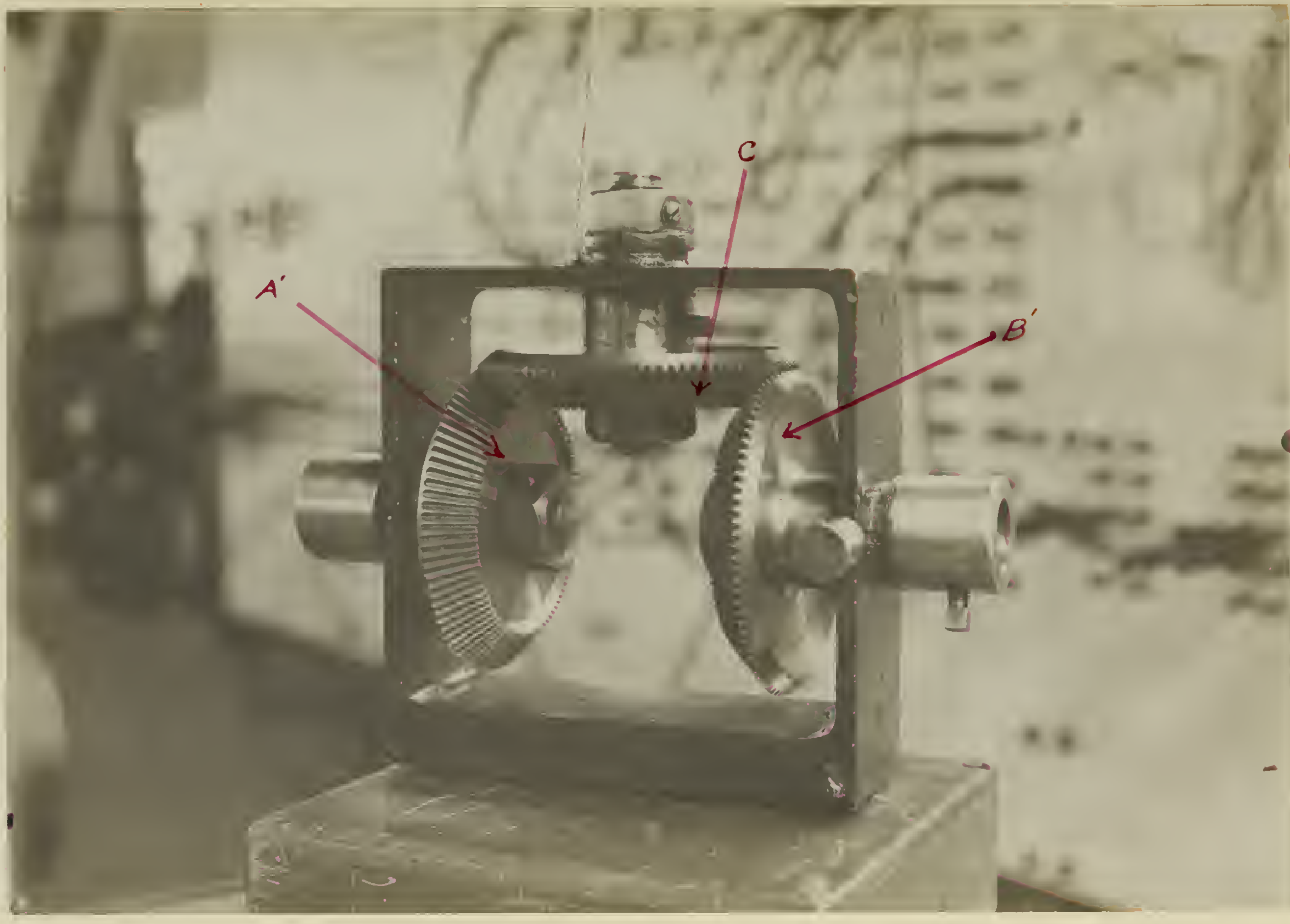


PLATE I

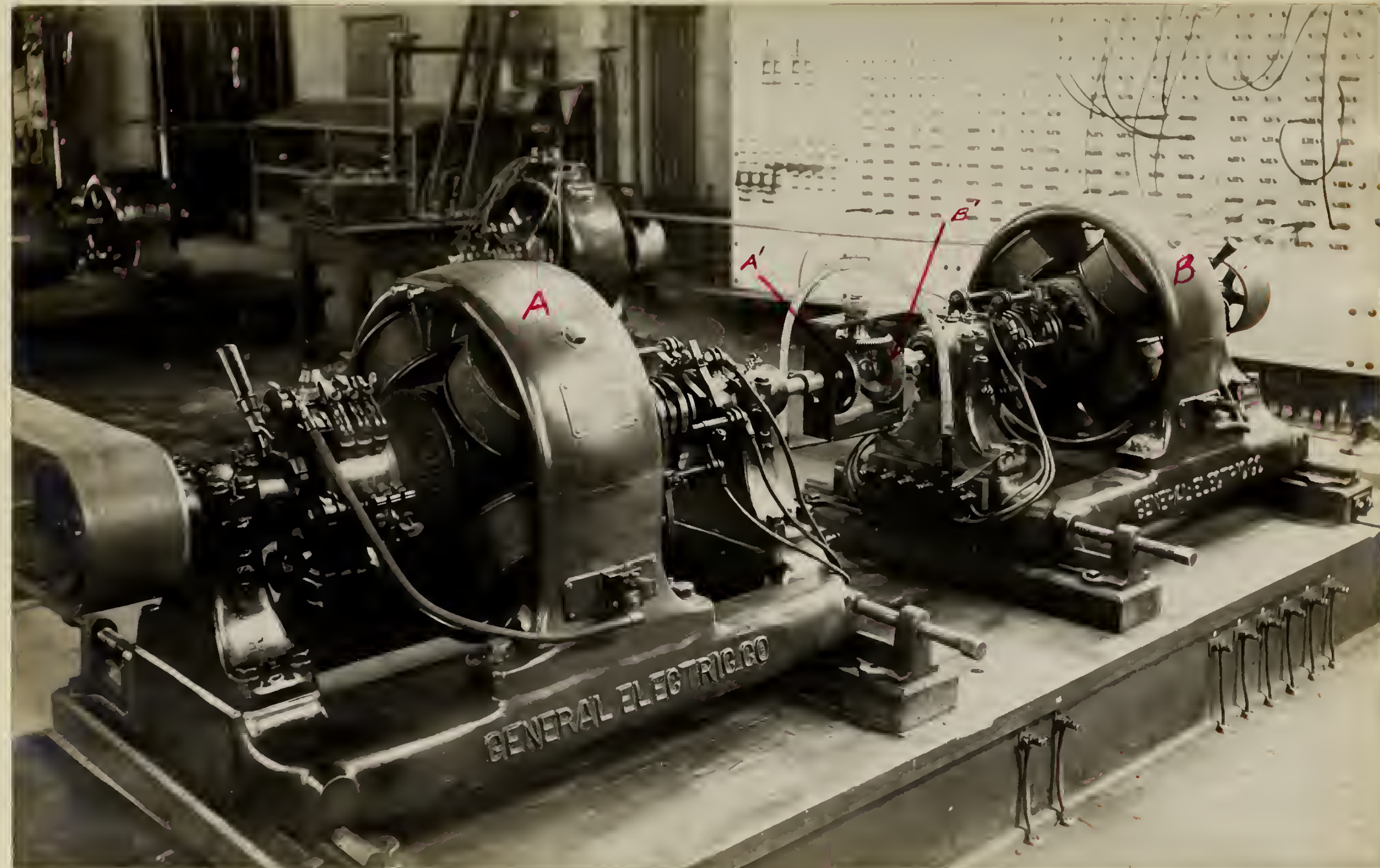


PLATE II

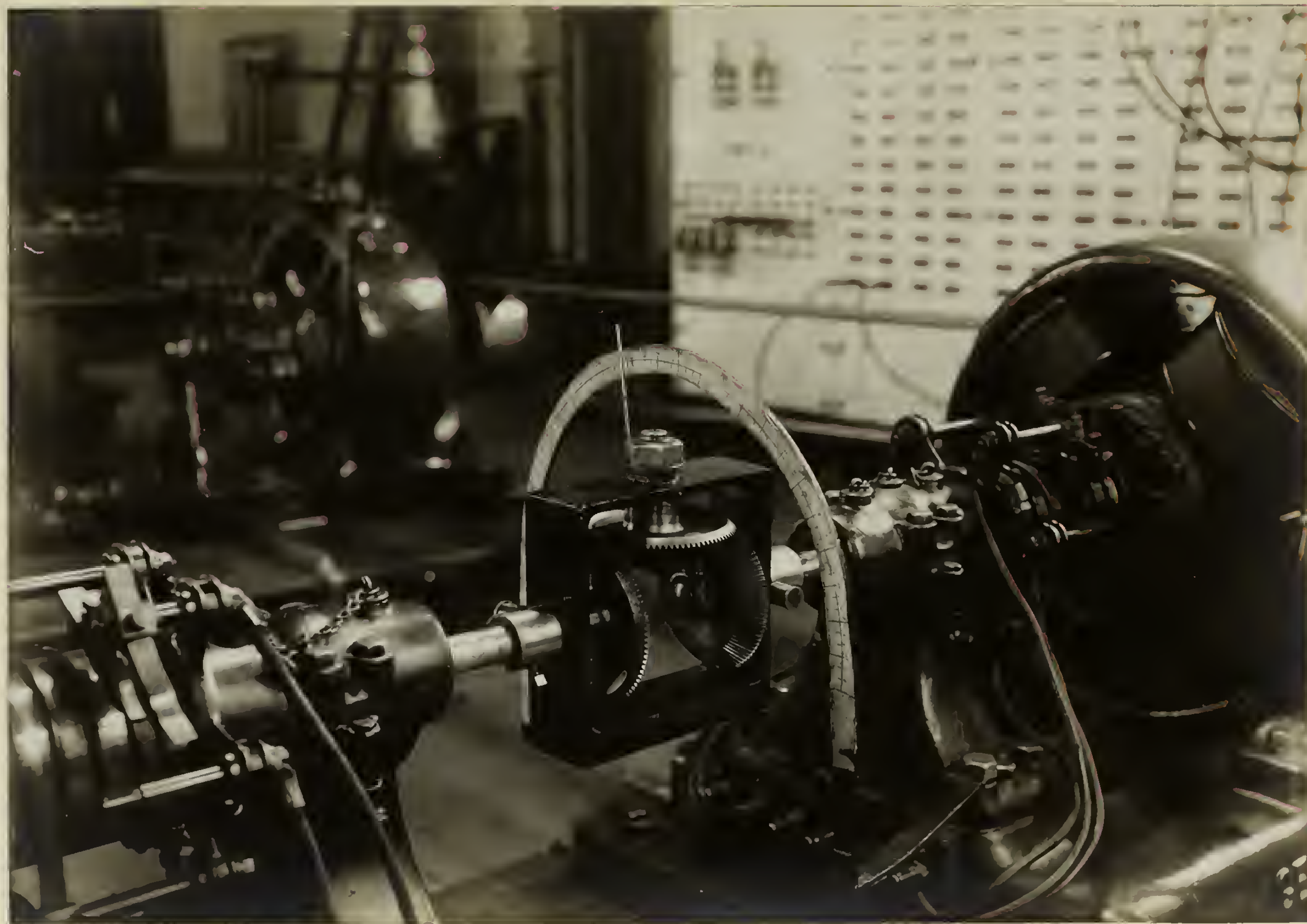


PLATE III

If at some instant the speed of the two machines is the same, then the pointer is standing still, but if one machine begins to lag behind the other, the angular lag may be measured directly by means of the pointer, which is attached to the frame of the gearing and the scale as shown in Plate III. If one machine should lag, mechanically, ten degrees behind the other, the pointer would show a lag of five degrees, the ratio of the gears being such that the pointer registers only one-half of the amount that one machine may lag behind the other.

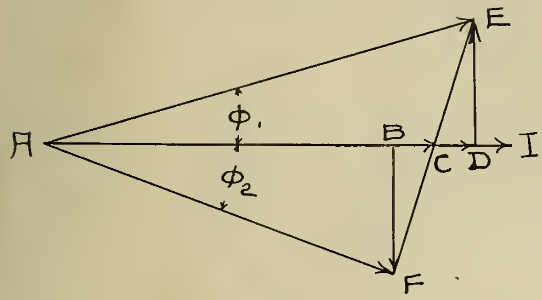
The scale just referred to consists of a semicircle divided into 360 equal parts so as to read in electrical degrees rather than in mechanical degrees since each revolution of the armature of a four pole machine corresponds to 720 electrical degrees. The actual mechanical lag is of interest since it bears a certain fixed relation to the angular difference between the E. M. F. vectors of the generator and the motor.

THEORY.

The fundamental notion of successful synchronous operation is that the E. M. F. waves of the machines which are running in synchronism must be of similar form and that they must vary simultaneously, i. e., they must reach their maximum and minimum values at approximately the same instants. In this thesis it will be assumed that the E. M. F. waves of the machines are of sine wave form and that the value of these electromotive forces may be represented by vectors rotating in space, the length of these vectors being proportional to the instantaneous values of the electromotive forces generated by the machines.

Let us consider any given coil on the armature of an alternator. Then, for every position of the coil, as the armature revolves, there is a corresponding value of electromotive force in the coil, assuming constant field excitation and speed. In other words, every time this coil passes the leading pole tip of a certain north pole, the E. M. F. induced in the coil is of exactly the same value. From these considerations, we see that there is a direct relation between the E. M. F. vector of a machine and the mechanical rotation of its armature. As stated before, if two similar machines are exactly in step, their E. M. F. vectors are rotating together and they are always of equal length. Under these conditions the armatures rotate together as though they were keyed upon the same shaft. However, if one armature lags behind the other, one of their E. M. F. vectors must also lag behind the other. For two pole machines, if one armature lags behind another by five degrees, then there will be a difference of five electrical degrees between their E. M. F. vectors and at any instant the projection of one of these vectors upon a diameter of the circle described by its extremity will be longer than the other, assuming that the two machines are being operated as generators. By taking advantage of these facts, it is possible to measure the amount which the electromotive force vector of one machine lags behind that of another when two machines are running in parallel, providing the mechanical construction of the two machines is identical and that they are connected together by means of a differential gearing as previously described.

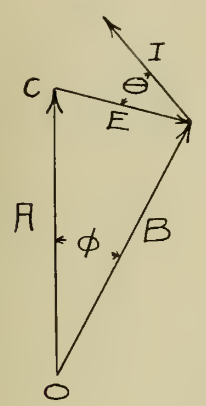
Representing the E. M. F. vectors diagramatically the relation between the angle as measured by the gearing and the angle between the Generator terminal voltage and motor induced voltage will be shown.



Let AC equal the generator terminal voltage to scale. To find the length and position of the induced generator voltage with respect to I, the IR drop CD and the reactance drop ED are added vectorially to AC. This gives AE the Generator induced voltage making an angle ϕ_1 with AC. To find the position of the motor induced voltage the motor IR drop CB and the reactance drop BF must be subtracted from AC leaving AF at an angle ϕ_2 with AC. Angle ϕ_2 is not exactly equal to ϕ_1, as is evident from the construction, but is so nearly so that for our purposes it may be taken equal to it. The angle as measured by the gear is EAF which is equal to or 2 ϕ_1. The angle between the motor induced voltage and the generator terminal voltage is CAF or $\frac{2\phi}{2}$ which is equal to ϕ .

When two similar machines are operating as generators in parallel and have their fields equally excited, their vectors are coincident or very nearly so, any small angle between them being due to the difference in driving force of the machines. When the two vectors are exactly equal and coincident, there will be no current flowing between the machines, assuming that they are thrown on the bus bars without load. As soon as the vectors are not coincident there will be an electromotive which is the

vector difference between the two E. M. F. vectors acting upon the circuit, to produce a current known as the circulating or synchronizing current of the machines. It is this current that keeps the machines in step. If, now, the driving force of one machine is removed so that it operates as a motor, the E. M. F. vector of the motor, (B, of the accompanying figure) will lag behind A of the generator by an angle ϕ .



This will cause an electromotive force E to act upon the circuit and produce a current I which makes an angle θ with E. The angle may by determined directly by means of the differential gearing, or from electrical measurements as follows: A is the terminal voltage of the generator as measured by a volt-meter and is laid off to some convenient scale. Since the motor field current is known, its induced pressure may be determined from its magnetization curve. Using the same scale, this value is used as a radius to strike off an arc of which O is the center. If, now, the value of E be known, an arc may be struck off with the point c as a center and E as a radius. The intersection of these two arcs will determine the location of B and also the value of ϕ .

Since E causes the current I to flow, it may be considered as a voltage, independent of A or B, which is impressed upon a circuit consisting of the resistance of the connections of the two machines to the bus bars and the bus bars themselves,

and the inductance and resistance of the motor. If we let z equal the total impedance of this circuit then E is equal to the product of I and z, both of which may be determined quite accurately. By this means ϕ can be determined and should check with the angle as measured by the differential gearing.

It is to be noted that Prof. Brooks, in his theory, assumed that the impedance of the generator should be included in the impedance z which must be overcome by E to produce the current, I, while in obtaining the true power factor of the circuit, cos θ , he used only the impedance of the motor and leads. The same conditions should be assumed in determining the value of the impedance z which E must overcome to produce I. Since the field current is adjusted to maintain a constant terminal pressure at the generator, the armature reactions and the armature drop in the generator need not be considered. We have only to consider the motor circuit. The generator serves as a reference machine.

It is upon these few simple principles that the theory of our diagram is based. For a more complete discussion and the mathematical proofs of the theory of the Circle Diagram as applied to synchronous machines, reference should be made to Professor Brooks' paper.

Throughout this paper, his notation and symbols will be used as far as possible. Any additional theory necessary for a thorough understanding of our methods of working will be included in the discussion of diagrams.

METHOD OF OPERATION AND DESCRIPTION OF TESTS.

Two 220-volt D.C. shunt motors were used to bring the machines up to synchronous speed, and as soon as they were synchronized, the power was cut off one of the motors which became a D. C. generator driven by one of the A. C. machines acting now as a synchronous motor. Power was supplied to this motor by the other A. C. machine which was operating as a generator. This generator was driven at normal speed by the other D. C. machine which continued to operate as a motor. The D. C. generator was used as a load for the synchronous motor in the various tests. A diagram of the electrical connections is shown in Plate IV.

TEST A. After the two machines were brought up to speed and synchronized, with both machines running at normal speed as generators under no load, the position of the pointer was noted, it being assumed that under these conditions, the two vectors were coincident. The load now placed upon one of the synchronous machines, as explained, was kept constant and normal synchronous speed maintained. The E. M. F. at the terminals of the generator was also kept constant at 110 volts. The field of the motor was varied from .75 ampere to 3 amperes, excitation being about 1.5 amperes for 110 volts. For each of the different values of field current, the following readings were taken; current and watts imput to the motor, the terminal voltage at the motor, and the position of the pointer, on the gear device. Five such sets of data were collected for different motor loads.

TEST B. This test was made in exactly the same manner as the one previously described, excepting the terminal

pressure of the generator was kept at 75 volts instead of 110 volts. This lower voltage was used in order that the machines might be thrown out of step without having such heavy currents as are produced when the machines are thrown out on a voltage of 110.

TEST C. The general method of conducting this test was much the same as that described for Test A. In this instance, however, the motor field was kept constant and the load on the motor was increased from nearly no load (the motor running the D. C. machine on open circuit) to full load. The same readings were taken as in the previous tests. Three different values of field current were used, thus making three distinct tests.

TEST D. This test was carried on in exactly the same manner as the first one. A rheostat was put in series with the generator and motor and various values of resistance thus introduced into the circuit. These resistances were used to bring about conditions similar to those where the length of transmission lines introduces considerable resistance between the power plant and the substation or other point where the motor or synchronous convertor may be operated. Besides the readings taken in Test A, readings of the power output of the generator were taken also.

TEST E. These tests were made with various values of resistance in series with the generator and motor. For each case, the resistance and armature current were kept constant to give a constant I^2R loss. To maintain these conditions, the motor field current and its load were varied. The same readings as those taken in Test D were taken for these tests.

DISCUSSIONS OF DIAGRAMS AND DATA.

METHOD OF CONSTRUCTION. (See Plate V)

From O, OA is laid off to scale equal to the generator terminal pressure. This position OA is the reference position of the vectors, it being assumed that they are together when both machines operate as generators on no load. From the motor field current the value of the induced motor pressure is found from the magnetization curve and laid off from O (Vector B) to the proper scale, the angle which it makes with vector A being taken from the measurement made by the differential gearing. This is repeated for the various angular positions of B, the extremity of B in the various positions being denoted by small circles, the loci of these points giving motor load curves. The center of the reference circle was determined by the intersection of two lines drawn from the extremities of A, each making an angle θ with A. The angle θ is determined by the power factor of the circuit, cos. θ which is equal to $\frac{R}{z}$, R and z being the resistance and impedance of the whole circuit external to the generator. This center c is used to strike off arcs of circles which best fit the points representing the extremities of B, these arcs representing the theoretical form which the load curcurves should take. This gives at a glance the relation between the theoretical curves and those obtained from the results of tests. In order to check the measurements as made by the differential gearing, the value of E (equal to Iz) was calculated from the elctrical measurements taken in Tests $A_{(a,b,c,)}$, $B_{(a)}$, $C_{(ab)}$ and the position of B and also the value of ϕ determined. The

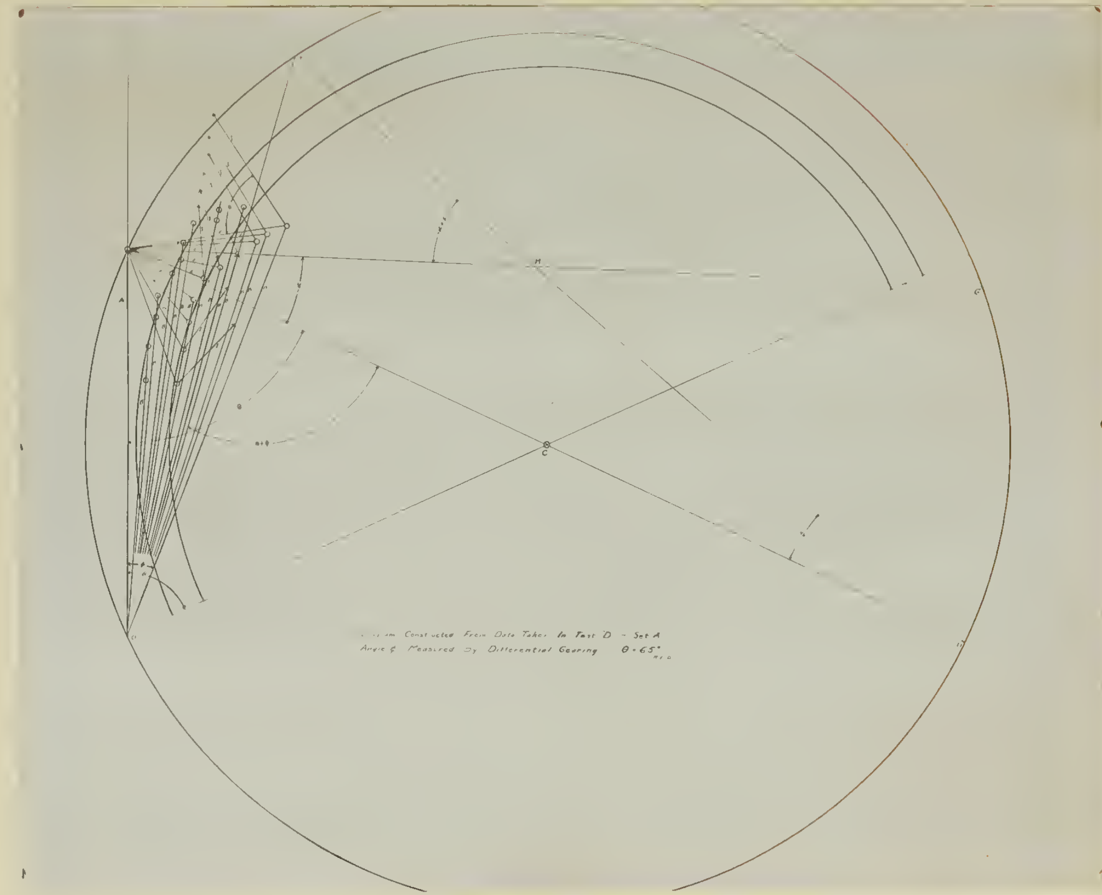

Constructed From Data Taken In Test D - Set A
Angle Measured By Differential Gearing θ = 65°

PLATE V

general method of constructing these diagrams was explained in the discussion of the theory.

In the diagram of Plate V the current I was laid off to scale making the angle θ with E. For all diagrams excepting Plate V the scale adopted for volts is; 1 mm = 1 volt. For Plate V, 2 mm. = 1 volt. 1 mm. = 1 ampere.

CONCLUSIONS.

The results of the tests made show that this Circle Diagram is of decided value in studying the interaction of synchronous machines and that its theory agrees as fully with experimental results as does much of the Alternating Current theory that has been accepted for years without question.

While only a limited number of tests have been made, these few have been made carefully and have been repeated a sufficient number of times to make it reasonably certain that the results submitted are as accurate as could be ordinarily obtained in experimental work of this nature. The largest errors probably occurred in determining the angle ϕ by means of the gearing, and in reading the ammeter which was used to measure the current, I. In cases of unstable operation where the machines were about to fall out of step, it is possible that the error in ϕ may have been as great as 10%. The error in reading the ammeter probably did not exceed 5%, but at times it was undoubtedly as great as this.

A study of Plate V, which is a typical diagram, shows that for the larger values of ϕ, the vector B fails to reach

the theoretical load curve, i. e., the induced motor voltage is really less than that assumed in the Circle Diagram theory. The authors have not been able to find any satisfactory explanation of this falling off of B from a true circle. Since the value of B is taken from the magnetization curve, no armature reactions in the motor are taken account of. However, a consideration of armature reactions will show that B must be less than the value taken from the magnetization curve, so we must look elsewhere for an explanation of this variation of actual conditions from those determined from theoretical considerations only. For the ordinary range of operation the load curve will not vary greatly from a true circle and no large error will be made in assuming that the locus of B is a circle.

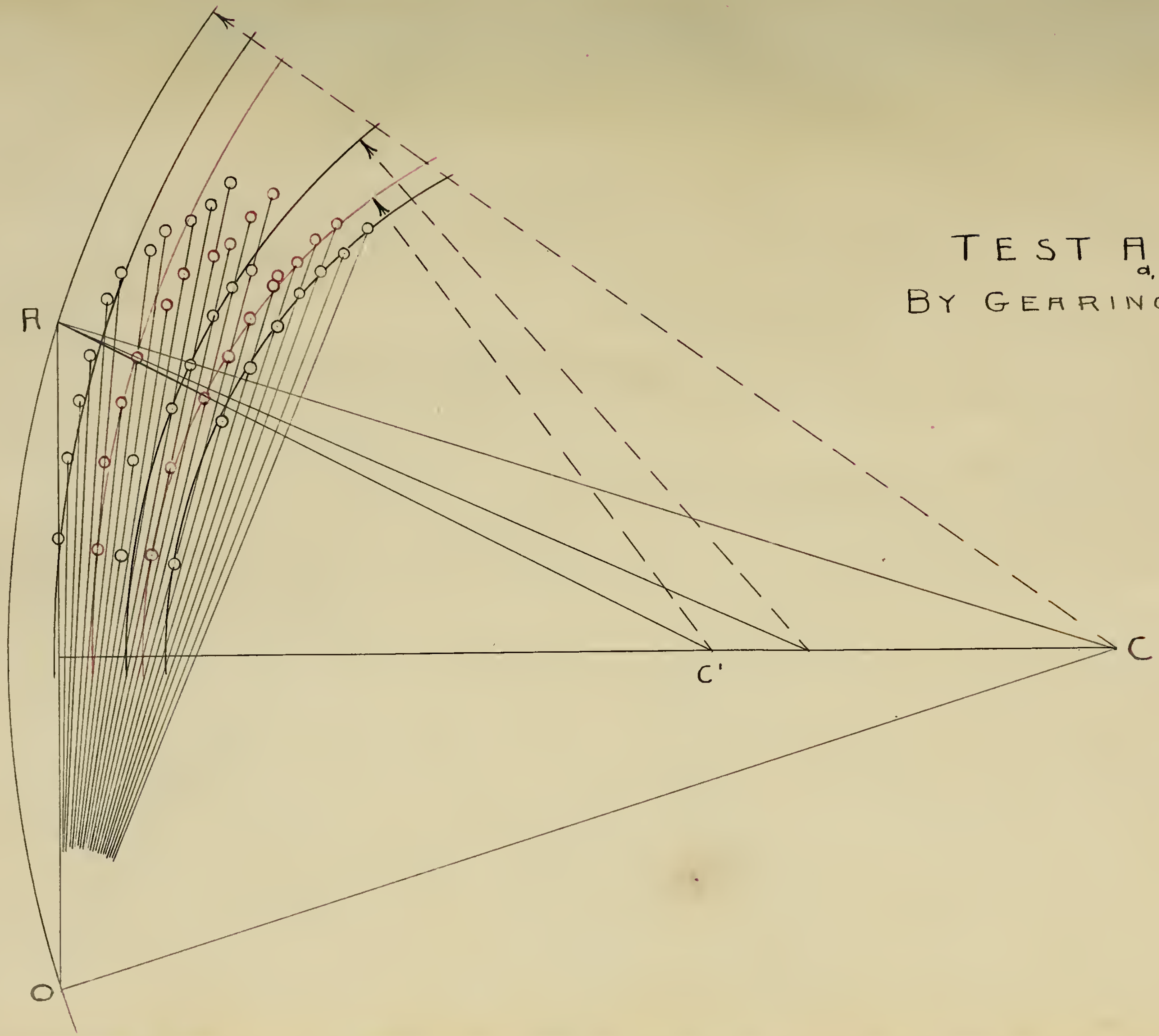
In the Circle Diagram the impedance of the armature is considered constant for all loads and the center of the reference circle is determined with this assumption as a basis. Our present knowledge of this subject leads us to believe that this is not true. If the armature inductance does vary, then the various load curves will not have the same center. The diagrams plotted from the data taken in Test A indicate that the impedance of the armature is not constant. The shifting of the center is quite likely due to the change in the armature inductance of the motor.

The diagrams plotted for a constant I^2R loss give a locus for B which does not check well with the theoretical locus of B.

A more extended series of tests would undoubtedly show, as these already presented, have done, that the Circle Diagram is of value and that for all ordinary conditions it may be used to predetermine the behavior of synchronous machines. It is doubtless one of the simplest diagrams yet worked out for the study of synchronous operation.

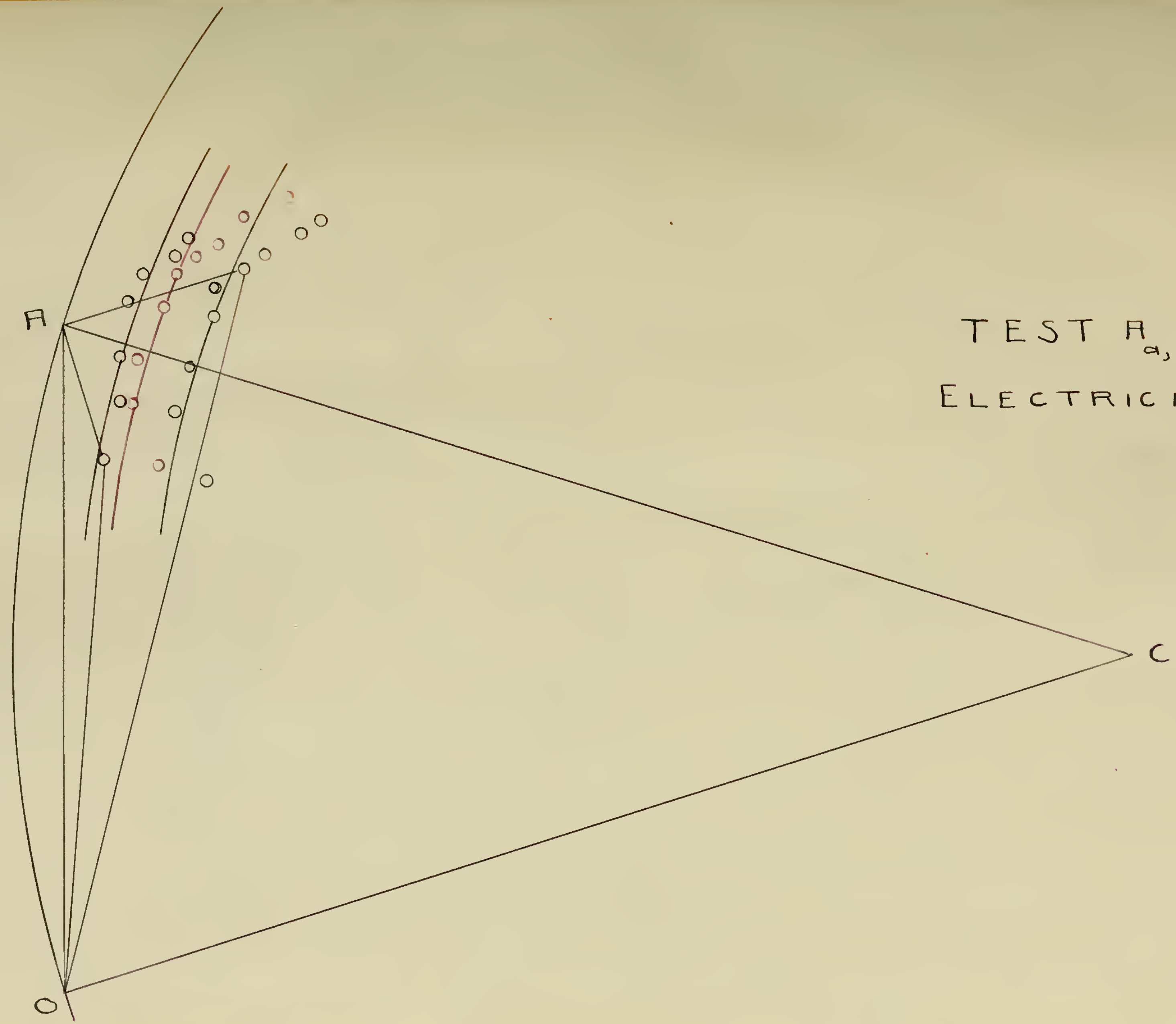
TEST $A_{a,b,c,d,e.}$

BY GEARING.

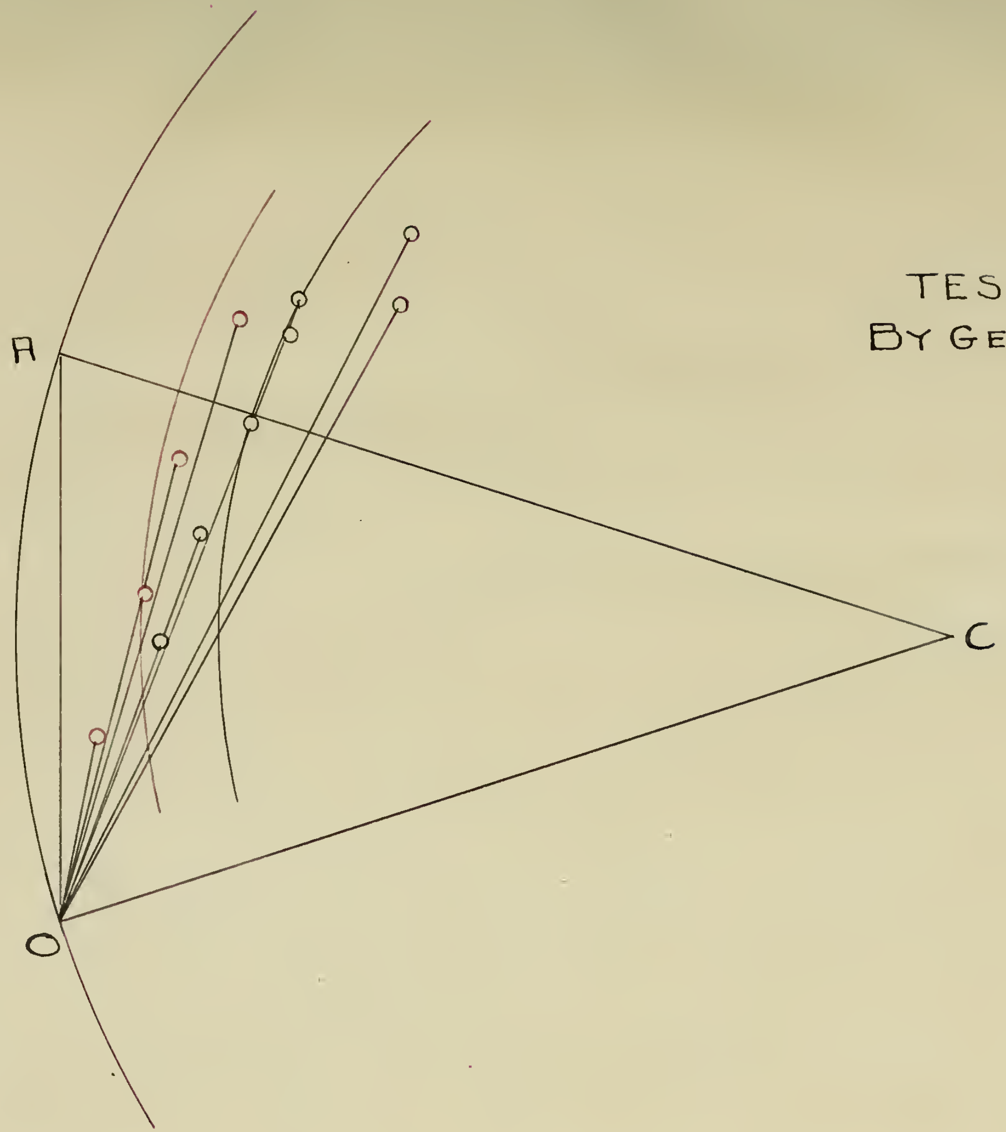


TEST $A_{a,b,c}$.

ELECTRICALLY.

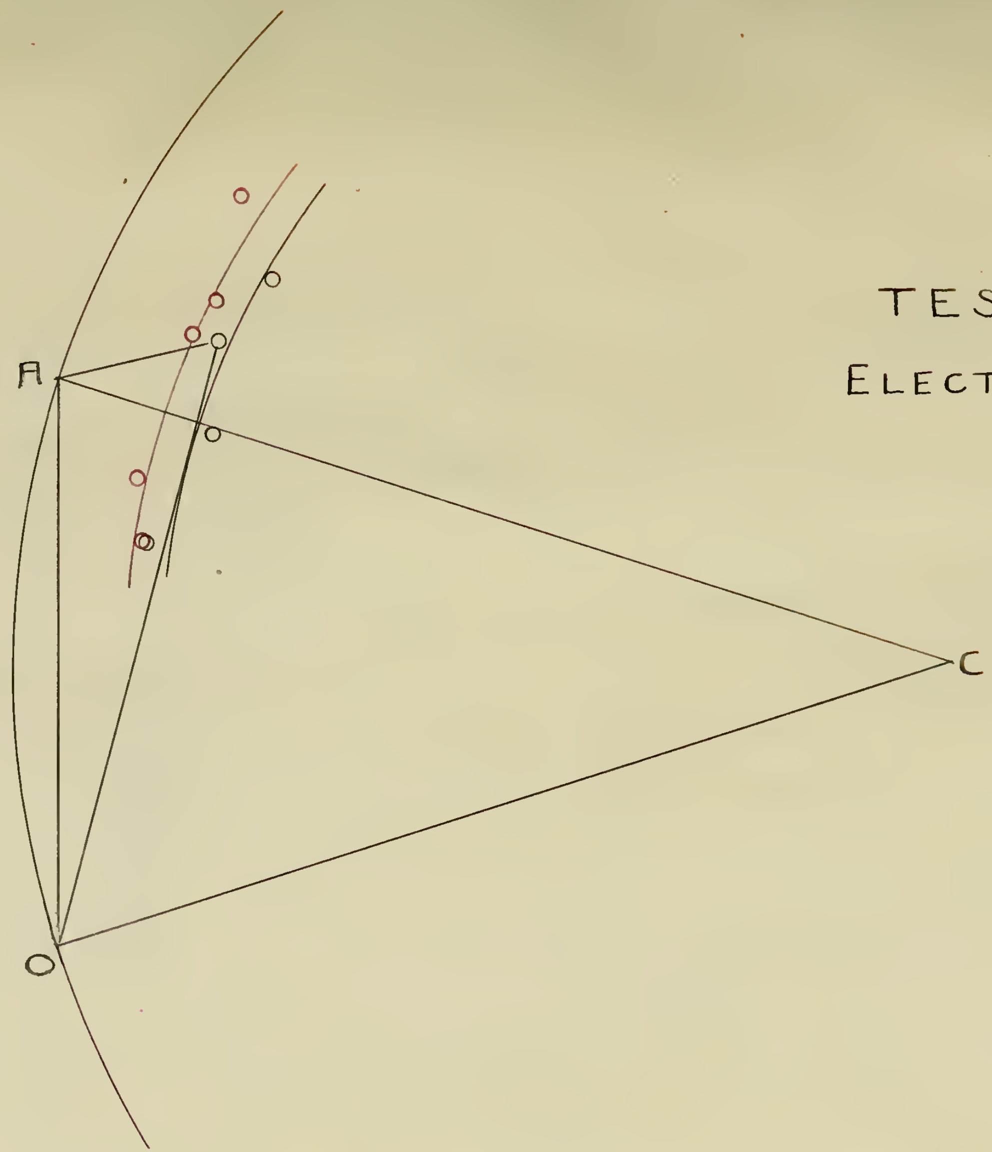


TEST B_a
BY GEARING



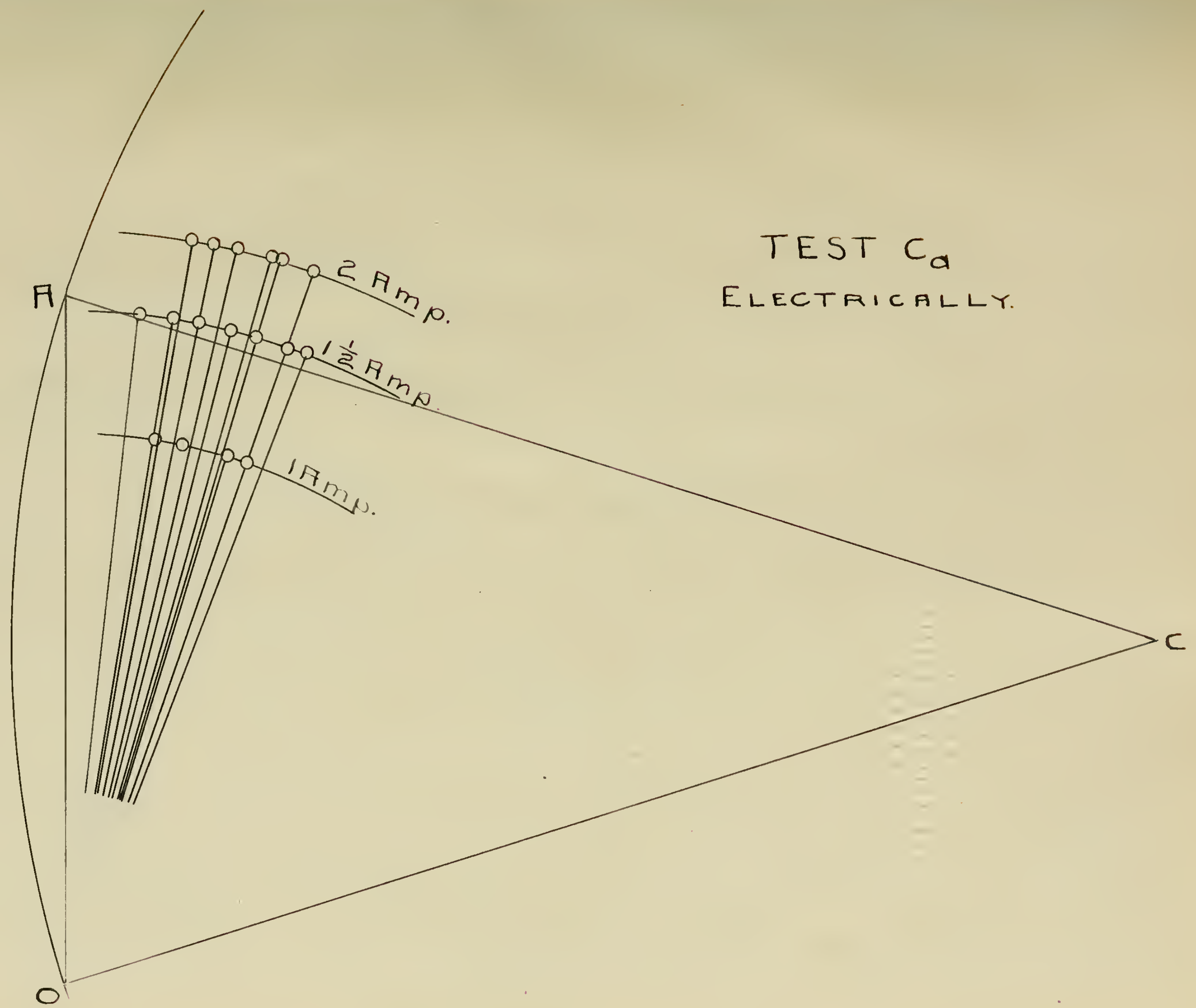
TEST B_a

ELECTRICALLY



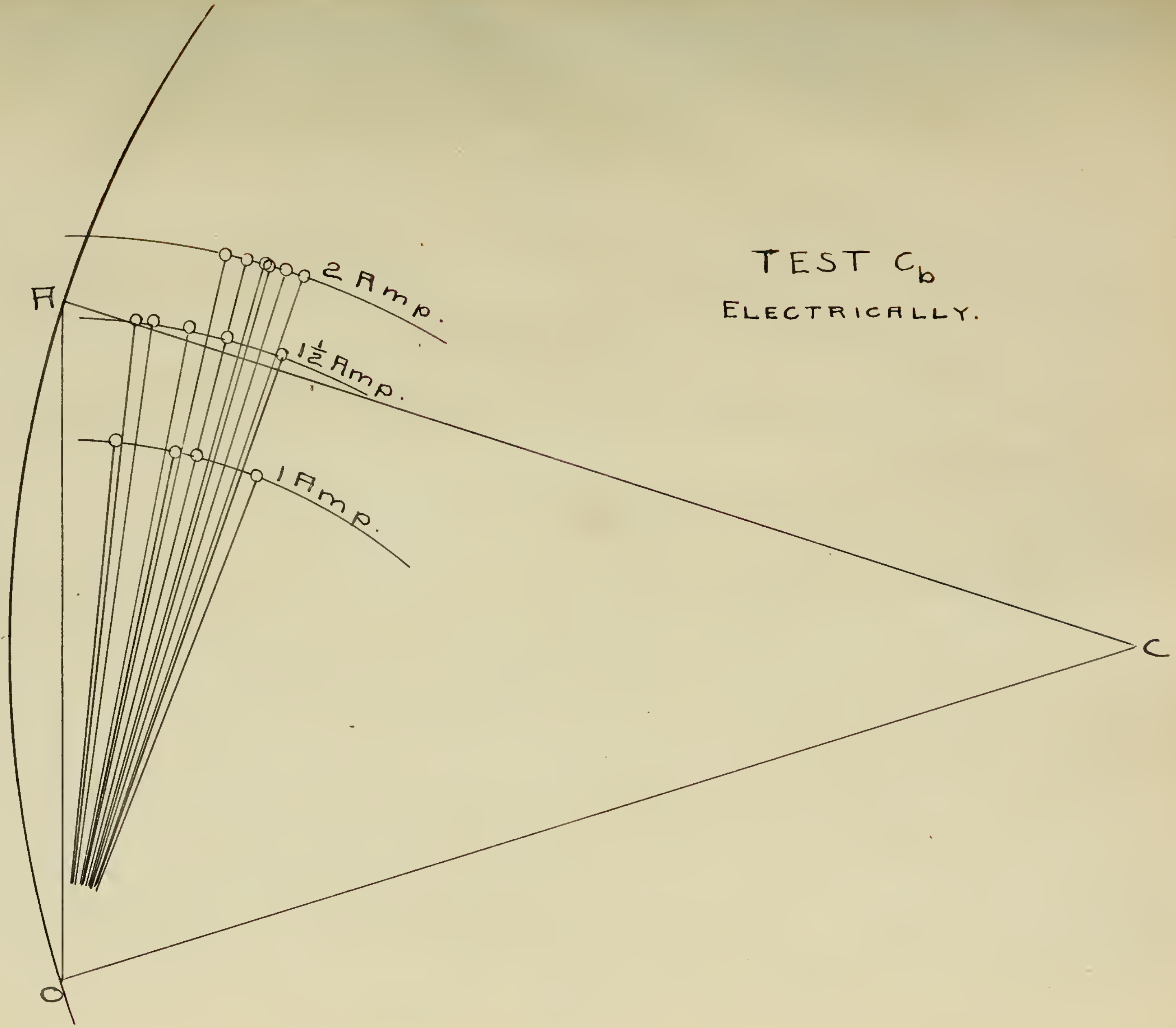
TEST C_a

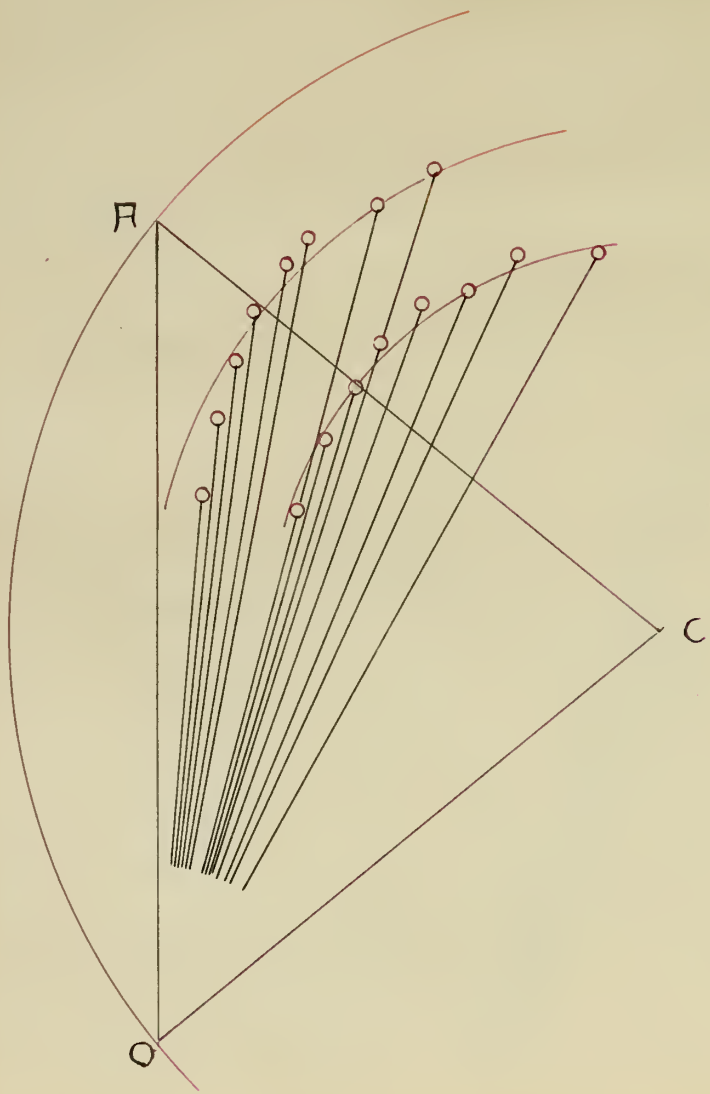
ELECTRICALLY.



TEST C_b

ELECTRICALLY.

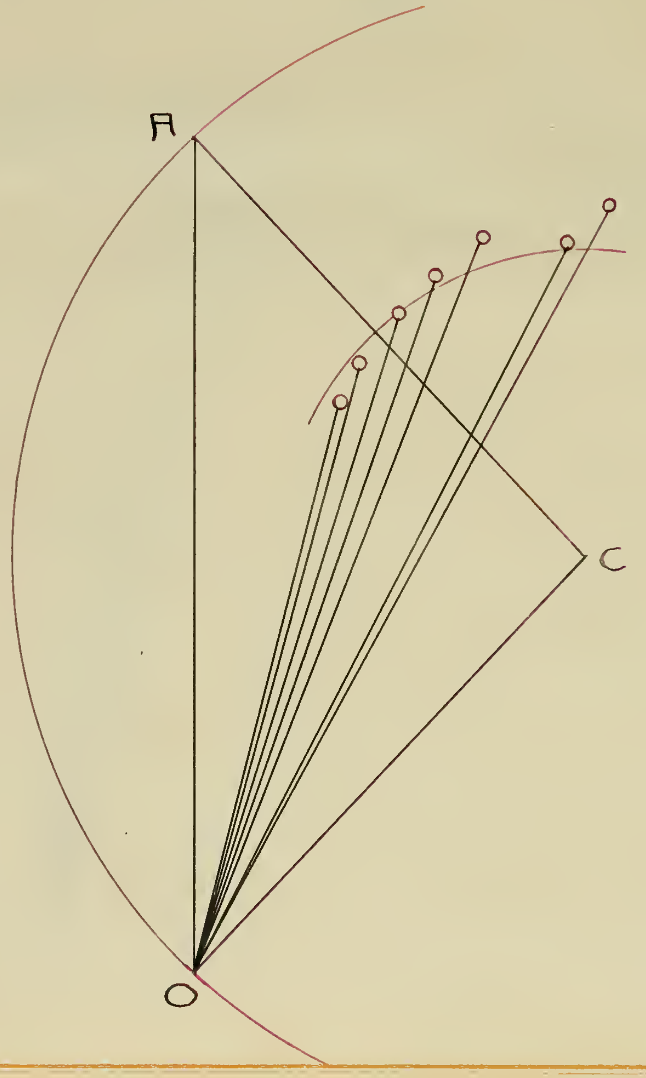




TEST D_b

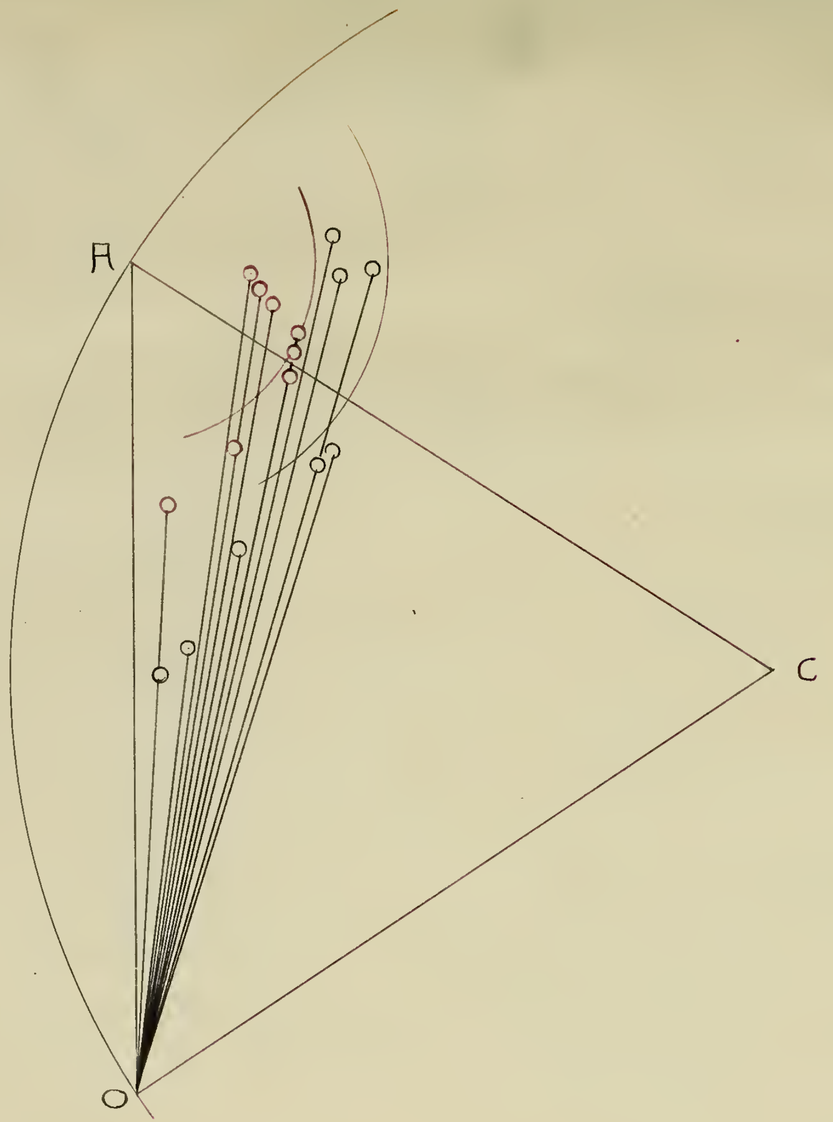
BY GEARING.

TEST D_c
BY GEARING.

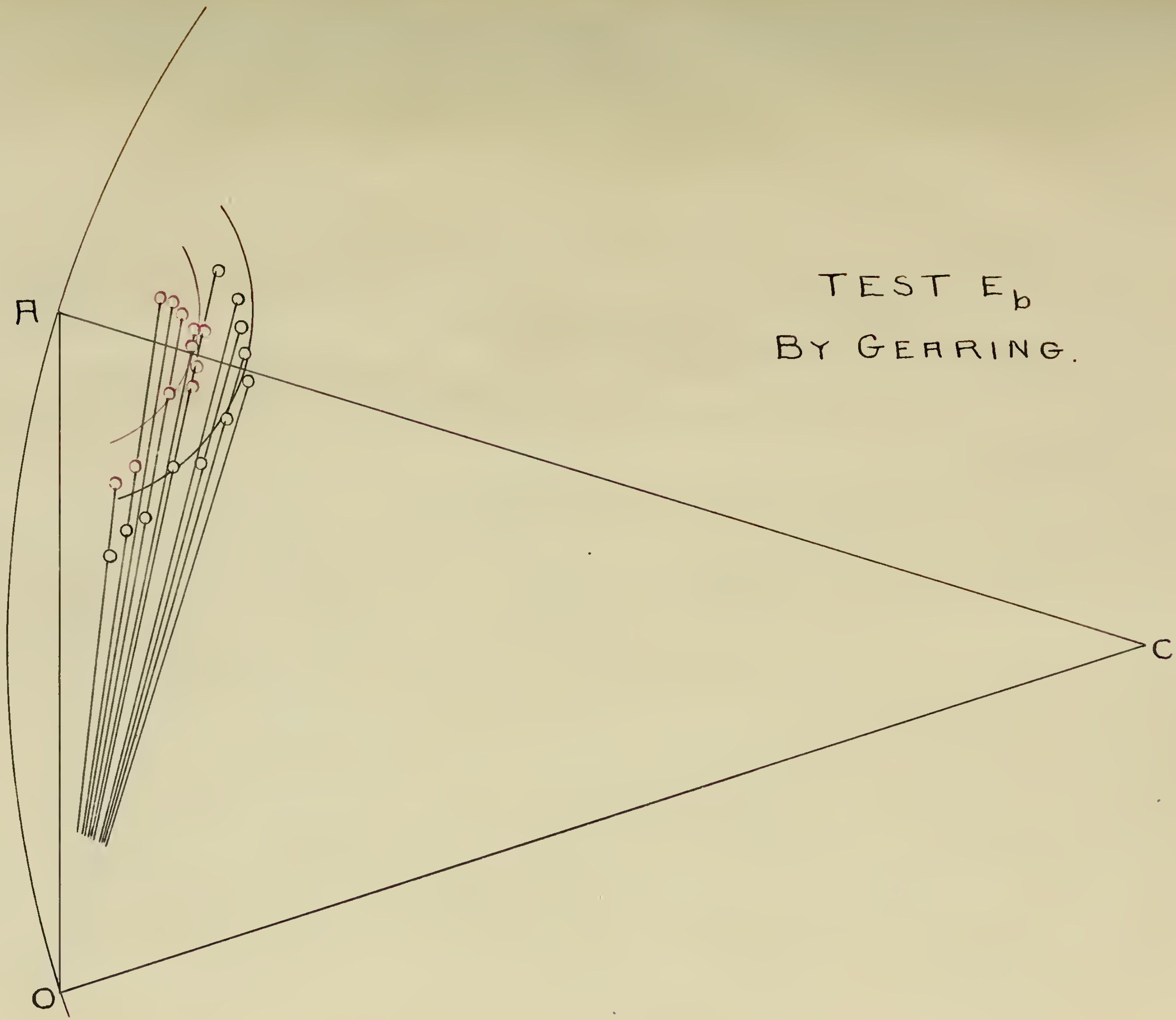


TEST E_d

BY GEARING.

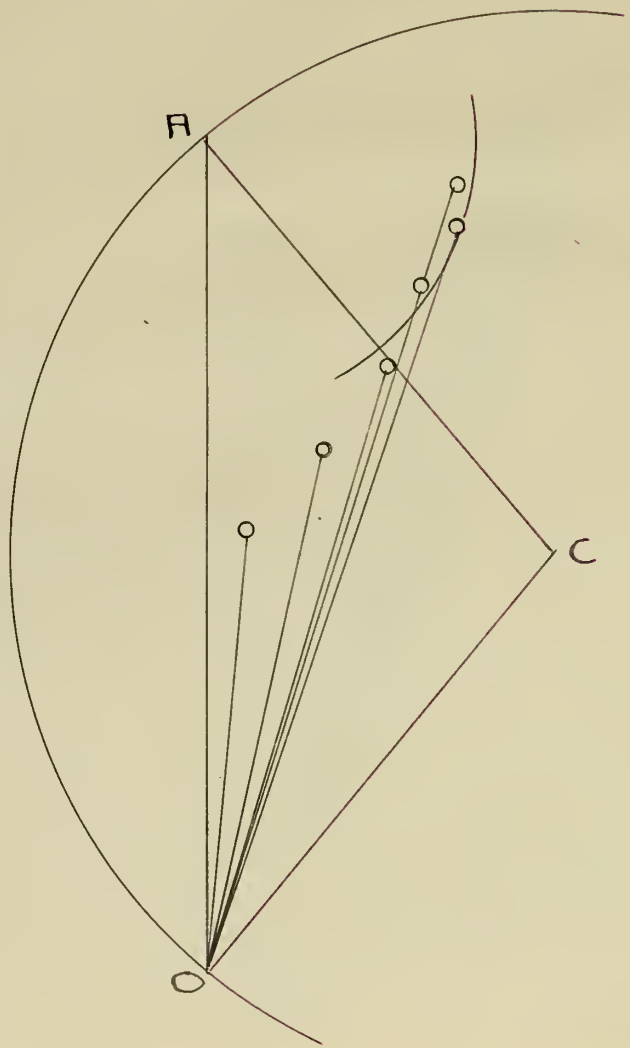


TEST E_b
BY GEARING.

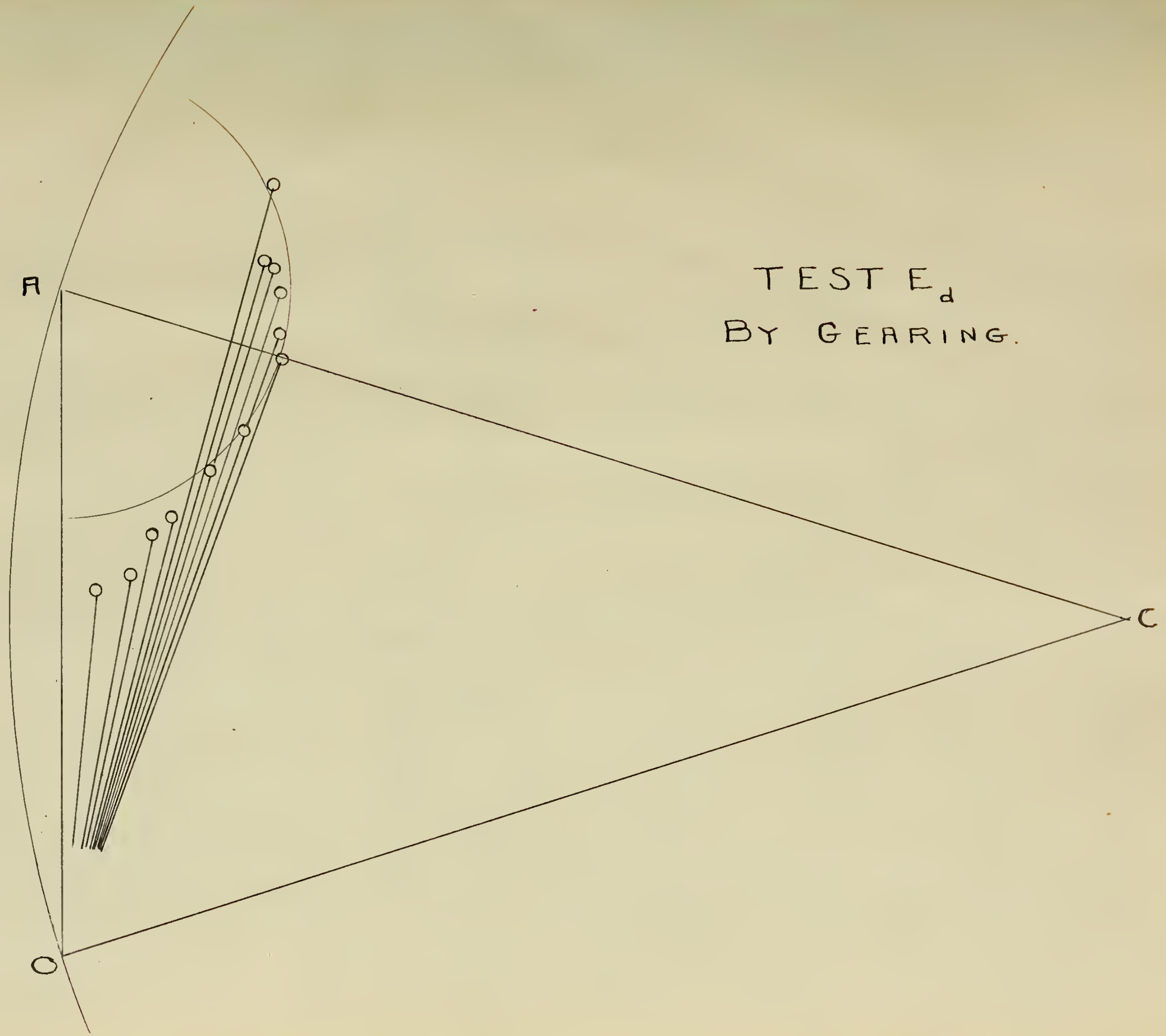


TEST E_c

BY GEARING.



TEST E_d
BY GEARING.



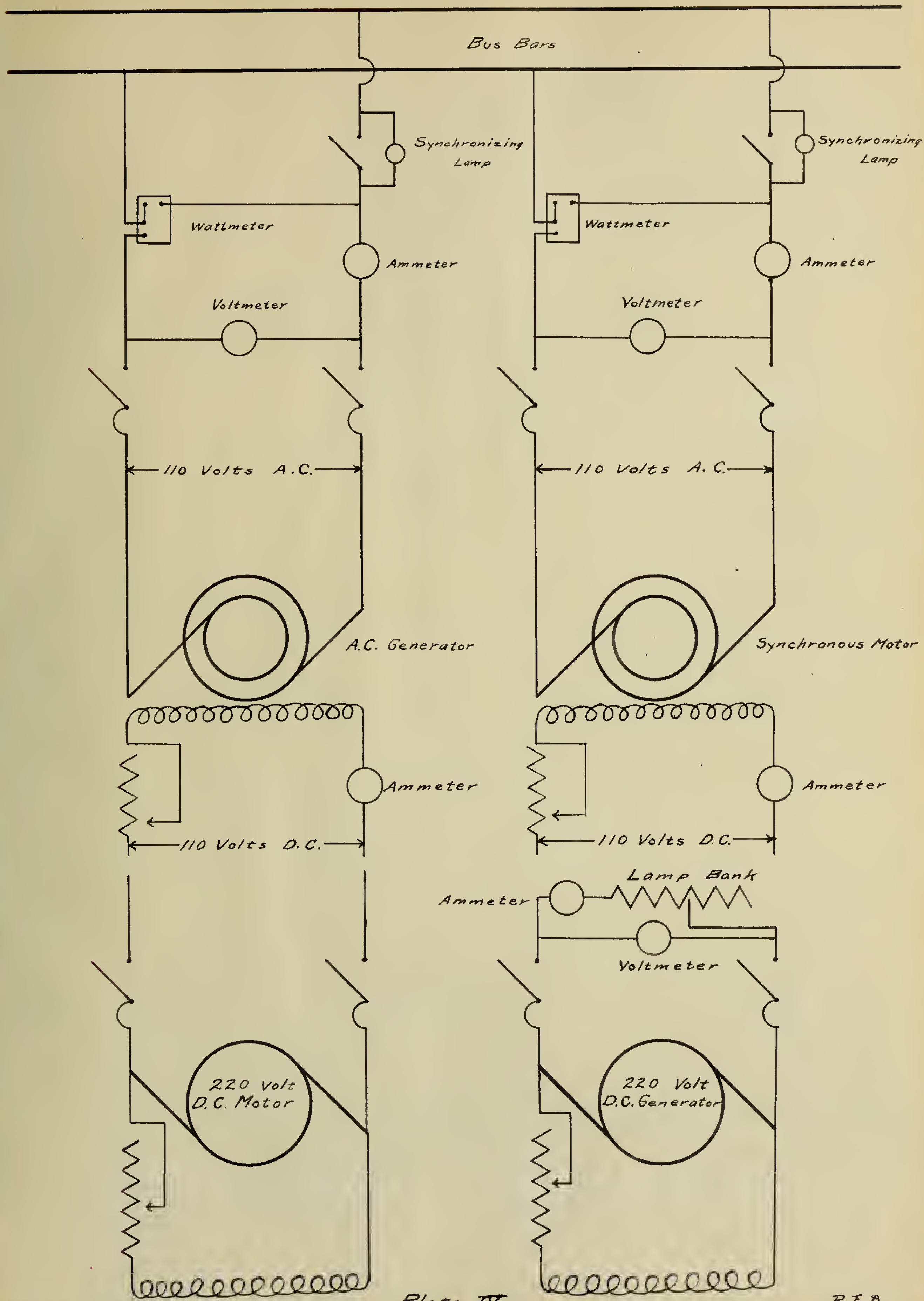
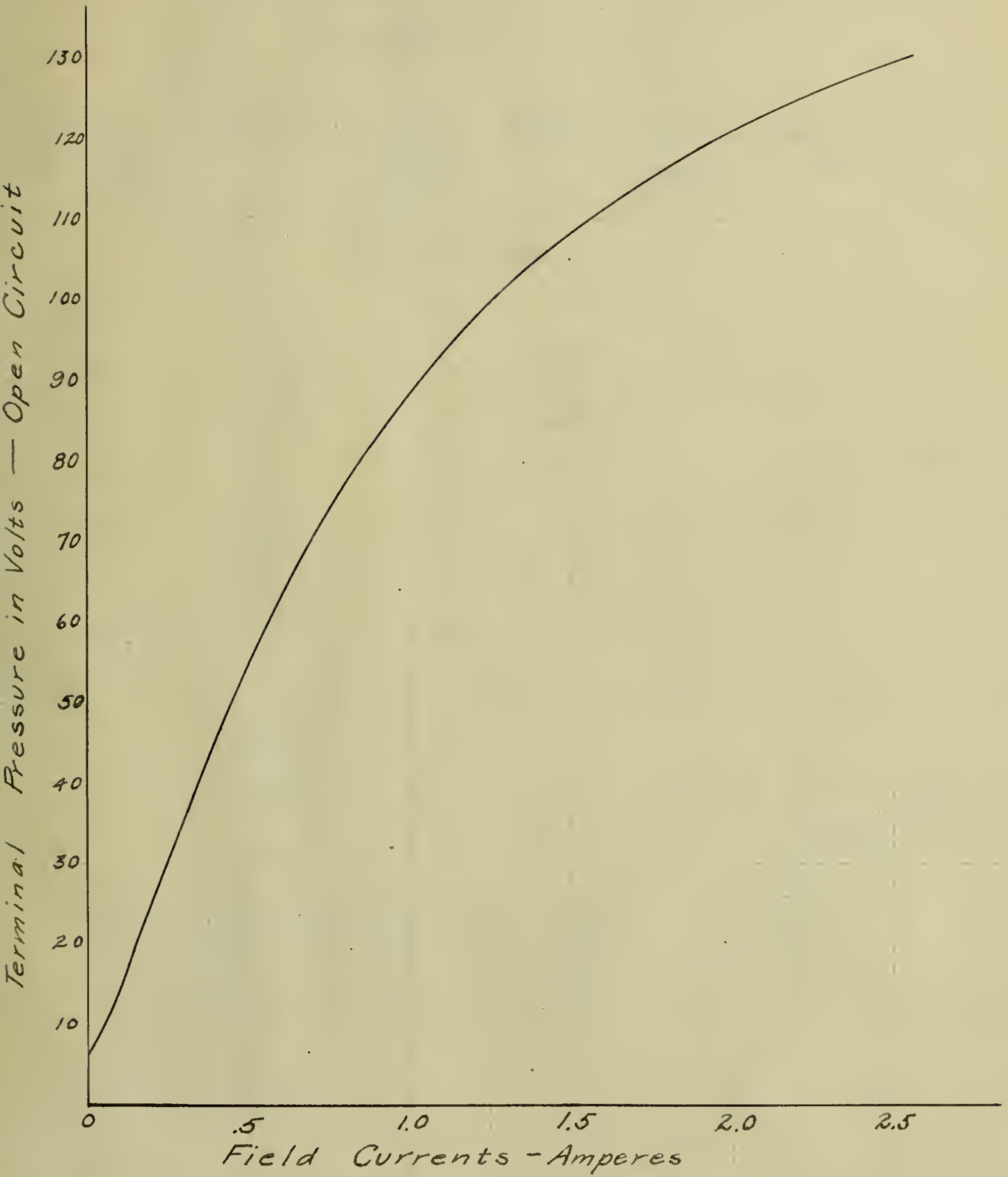

Bus Bars
Synchronizing Lamp
Synchronizing Lamp
Wattmeter
Wattmeter
Ammeter
Ammeter
Voltmeter
Voltmeter
110 Volts A.C.
110 Volts A.C.
A.C. Generator
Synchronous Motor
Ammeter
Ammeter
110 Volts D.C.
110 Volts D.C.
Lamp Bank
Ammeter
Voltmeter
220 Volt D.C. Motor
220 Volt D.C. Generator

Plate IV

R. E. D.

Magnetization Curves For G.E. Rotary No. 28366



TEST A_a.

Constant Generator Terminal Voltage-110 Volts.

Varying Motor Field.

Motor Terminal Voltage.	Armature Current.	K.W. Input Motor	Motor Field.	Angle by Gears. Degrees	Induced Motor Voltage.	IZ.
		As Generators-----200				
106	49.0	1.9	.76	200	75	30
106.5	37.5	1.8	1.00	199	88	23
106.5	25.5	1.77	1.20	198	97.5	15.6
107.5	16.5	1.75	1.40	197	105.0	10.1
107.0	16.5	1.75	1.72	196	114.0	10.7
108.0	24.5	1.85	1.90	195	119.0	15.0
106.5	34.5	1.95	2.10	193	123.0	21.2
107.0	40.0	2.0	2.30	192	126.0	24.5
107.0	52.0	2.25	2.50	190	129.0	31.9
107.0	59.0	2.5	2.70	189	132.0	36.1
107.0	70.0	2.6	3.00	188	136.0	42.8

Resistance Motor and Circuit, .184 ohms.

Impedance Motor and Circuit, .608 ohms.

$\theta = 72.1/2^{\circ}$.

TEST A_{bc}.

Constant Generator Terminal Voltage-110 Volts.

Varying Motor Field.

Motor Terminal Voltage.	Armature Current.	K.W. Input Motor.	Motor Field.	Angle by Gears. Degrees.	Induced Motor Voltage.	IZ.
			As Generators-----200			
103	50	2.55	.75	195	73	30.6
107	47.5	2.5	1.00	195	88	29.0
106	28.0	2.5	1.20	194	97.5	17.2
106.5	22.5	4.5	1.40	193	105.0	13.8
106.0	45.0	2.4	1.70	191	114.0	27.5
108	34.0	2.55	1.97	190	120.0	20.8
107	40.0	2.7	2.10	188	123.0	24.5
106.5	47.5	2.75	2.30	187	126.0	29.0
106	57.0	2.87	2.60	186	131.0	35.0
106	69.0	2.9	3.05	185	136.0	42.2
			As Generators-----199			
103.5	51.0	3.55	.72	192	72.5	31.2
104.5	57.5	3.75	1.02	192	88.0	35.2
104.0	39.0	4.00	1.20	189	97.5	23.8
104.0	36.5	3.95	1.40	188	105.0	22.2
104.0	40.0	4.05	1.70	187	114.0	24.5
104.0	42.0	4.1	1.90	186	119.0	25.7
103.5	51.0	4.1	2.1	185	123.0	31.2
104.0	57.0	4.2	2.3	183	126.0	35.0
103	69.0	4.25	2.62	181	131.0	42.2
103	75.0	4.4	2.81	180	134.0	46.0

Resistance Motor and Circuit, .184 ohms.
Impedance Motor and Circuit, .608
$\theta = 72\ 1/2^{\circ}$.

TEST A_{de}.

Constant Generator Terminal Voltage-110 Volts.

Varying Motor Field.

Motor Terminal Voltage.	Armature Current	K.W. Input Motor.	Motor Field.	Angle by Gears.	Induced Motor Voltage.	IZ.
		As Generators----199				
102	62	4.6	.75	188	73	38
101.5	50	4.65	1.00	187	88	30.6
102.0	44	4.7	1.28	186	101	27.0
101.0	42	4.6	1.50	185	108	25.7
102.0	47	4.65	1.75	184	115	28.8
102.0	53	4.75	2.00	183	121	32.5
102.0	63	4.8	2.3	182	126	38.5
101.5	75	4.85	2.6	180	131	46.0
100.0	85	5.00	2.9	178	135	52.0
		As Generators-----201				
100.0	69	5.6	.75	185	73	42.2
101.0	56	5.5	1.20	184	79.5	34.3
101.0	51	5.5	1.45	183	107.0	31.2
100.00	53	5.5	1.75	182	115.0	32.4
101.0	60	5.5	2.00	181	121.0	36.7
101.0	67	5.5	2.25	180	125.5	41.0
101.0	75	5.6	2.55	179	130.0	46.0

Resistance Motor and Circuit, .184.

Impedance Motor and Circuit, .608.

$\theta = 72\ 1/2^{\circ}$.

TEST B_a.

Constant Generator Terminal Voltage-75 Volts.

Varying Motor Field.

Motor Terminal Voltage.	Armature Current.	K.W. Input Motor.	Motor Field.	Angle by Gears. Degrees.	Induced Motor Voltage.	IZ.
		As Generators-----200				
70	75	2.0	.2	185	25	45.7
70	37	2.0	.4	187	45	22.5
71	28	1.9	.6	187	63	17.0
71	30	2.0	.8	185	83	18.3
70	39	2.0	1.0	181	88	23.7
70	55	2.25	1.3	175	102	33.5
		As Generators-----198				
68.5	52	2.5	.35	182	40	31.6
69.0	41	2.45	.5	184	55	25.0
70.0	35	2.45	.7	183	71	21.3
70.0	36	2.5	.9	182	83	22.0
68.0	52	2.75	1.1	175	93	31.7

Resistance Motor and circuit, .184 ohms.

Impedance Motor and Circuit, .608 ohms.

$\theta = 72\ 1/2^{\circ}$.

TEST C_a.

Constant Motor Field. Varying Motor Load.

Generator Terminal Pressure-110 Volts.

Motor Terminal Voltage.	Armature Current.	K.W. Motor Input.	Motor Field.	Angle by Gears. Degrees.	Induced Motor Voltage.	Angle electrically.	IZ.
			As Generators--204				
107	32	2.00	1.00	200	88		19.5
105	35	2.75	1.00	197.5	88		21.3
104	45	3.75	1.00	194	88	194.5	27.5
104	50	4.5	1.00	192	88	192.0	30.4
100	60	5.5	1.00	190	88	187.0	36.5
98	65	6.5	1.00	187	88	185.0	39.5
			As Generators--204				
106	20	1.5	1.50	198	108	198	12
105	28	3.0	1.50	195	108	195	17
104	35	3.75	1.50	192.5	108	193.5	21.3
103	44	4.5	1.50	191	108	190.0	26.7
102	50	5.25	1.50	189	108	187.5	30.4
100	60	6.5	1.50	187	108	184.5	36.5
101	65	6.5	1.50	185	108	183.0	39.5
			As Generators--204				
106	35	2.0	2.00	195	121	194.5	21.3
104	40	3.0	2.00	192.5	121	193.5	24.3
104	47	3.75	2.00	190	121	190.5	28.6
102	55	4.5	2.00	188	121	188.0	33.4
101	57	5.0	2.00	187	121	187.5	35.0
102	65	5.5	2.00	186	121	184.5	39.5

Resistance, Motor and Circuit, .184 ohms.
Impedance, Motor and Circuit, .608 Ohms.

TEST C_b.

Constant Motor Field. Varying Motor Load.

Generator Terminal Pressure-110 Volts.

Motor Terminal Voltage.	Armature Current.	K.W. Input Motor.	Motor Field.	Angle by Gears. Degrees.	Induced motor Voltage.	Angle Electrically.	IZ.
		As Generators--------202					
107	35	2.25	1.0	198	88		21.3
105	40	3.0	1.0	197	88	196.75	24.3
104	50	3.5	1.0	194	88	189.5	30.4
102	55	4.75	1.0	192	88	187.5	33.5
100	70	6.0	1,0	187	88	181.0	42.6
		As Generators--------202					
109	20	2.0	1.5	198	108	196	12
107	25	3.0	1.5	196	108	194	15.2
104	35	4.0	1.5	193	108	191	21.3
102	45	4.75	1.5	190	108	188	27.5
100	60	6.0	1.5	187	108	183	36.5
100	60	6.0	1.5	187	108		36.5
		As Generators--------202					
107	35	2.25	2.00	195	121		21.3
105	45	3.0	2.00	192	121	198.5	27.5
105	50	4.0	2.00	190	121	187.5	30.4
101	55	4.75	2.00	187	121	186	33.4
101	60	5.5	2.00	186	121	184.5	36.5
100	65	6.0	2.00	186	121	183	39.5

Resistance Motor and Circuit, .184 ohms.

Impedance Motor and Circuit, .608 ohms.

DATA FOR PLATE V.

Constant Generator Terminal Voltage-110 Volts;

Constant Motor Load. Varying Motor Field.

Impressed Motor Voltage.	Motor Input K.W.	Current External Circuit. I.	Generator Output K.W.	Angle Diff, Gearing.	Induced motor Voltage. (Mag. curve)	Motor Field.	E =IZ.
		As Generators---204					
101	2.00	35	2.25	200	73	.75	22.3
102	2.00	29	2.10	200	83	..90	18.5
103	1.90	24	2.05	199	91	1.05	15.5
104	1.90	20	2.00	199	97.5	1.20	12.8
104	1.95	18	2.00	190	104	1.35	11.5
105	2.00	19	2.05	196	108	1.50	12.2
104	1.90	21	2.10	196	113	1.65	13.4
104	1.95	31	2.10	195	119	1.90	19.8
105	2.00	38	2.40	193	121	2.00	24.3
104	2.10	43	2.40	193	124	2.15	27.5
106	2.20	50	2.60	190	126	2.30	32.0
		As Generators----203					
96	4.20	47	4.70	190	73	.75	30.0
95	4.20	43	4.50	190	83	.90	27.5
98	4.20	40	4.50	190	91	1.05	25.5
98	4.15	40	4.40	189	97.5	1.20	25.5
98	4.15	41	4.50	187	104	1.35	26.2
98	4.20	45	4.50	185	108	1.50	28.7
98	4.30	50	4.75	183	113	1.65	32.0

DATA FOR PLATE V.

(Continued)

Impressed Motor Voltage.	Motor Input K.W.	Current External Circuit. I.	Generator Output K.W.	Angle Diff. Gearing.	Induced Motor Voltage. (Mag. curve)	Motor Field.	E = IZ.
98	4.35	57	5.00	183	118	1.85	36.4
98	4.50	65	5.50	182	121	2.00	41.2
97	4.70	75	6.00	180	125	2.20	48.0

$$\text{Cos. } \theta = \frac{R}{Z} = \frac{.27}{.639} = .423 \qquad \theta = 65^{\circ}.$$

TEST D_b.

Constant Motor Field. Constant Motor Load.

Generator Terminal Pressure-110 Volts.

Motor Terminal Voltage.	Armature Current.	K.W. Input Motor.	Motor Field.	Angle by Gears. Degrees.	Induced Motor Voltage.	Generator Output.	IZ.
		As Generators-----203					
96	29	2.0	.75	198	73	2.4	22
97	24	2.0	.90	197	83	2.2	18
99	20	2.0	1.05	196	91	2.05	15
99	20	2.0	1.2	195	97.5	2.15	15
100	23	2.0	1.35	193	104.0	2.25	17.3
100	27	2.05	1.5	192	108.0	2.4	20.3
99	40	2.25	1.75	188	115.0	3.0	30.0
98	53	2.4	2.00	185	121	3.75	40.0
94	72	2.75	2.3	175	129	3.25	54.2
		As Generators-----205					
87	40	3.5	.75	190	73	4.3	30
88	38	3.5	.90	189	83	4.2	28.6
90	38	3.5	1.05	188	91	4.2	28.6
90	40	3.5	1.2	187	97.5	4.25	30.0
90	43	3.6	1.35	185	104.0	4.45	32.4
90	50	3.7	1.5	182.5	108.0	4.8	37.6
88	60	3.75	1.75	180	115.0	5.5	45.2
87	70	4.0	1.95	176	120.0	6.5	52.7

Resistance of Motor and Circuit, .48 ohms.

Impedance of Motor and Circuit, .753 ohms.

$\theta = 50\ 1/2^{\circ}$.

TEST D_c.

Constant Motor Load. Varying Motor Field.

Resistance in Entire Circuit, .68 Ohms.

Constant Gen. Terminal Voltage-110 Volts.

Motor Terminal Voltage.	Armature Current.	K.W. Input Motor.	Motor Field.	Angle by Gears. Degrees.	Induced Motor Voltage.	Generator Output.	IZ.
		As Generators-----202					
83	39	3.4	.78	188	77	4.3	33.2
83	39	3.4	.90	187	83	4.25	33.2
84	39	3.4	1.05	185	91	4.4	33.2
85	42	3.4	1.2	183	97.5	4.5	35.7
86	47	3.5	1.35	181	104	4.8	40.0
81	59	3.5	1.5	175	108	5.9	50.0
81	20	3.8	1.75	174	115	6.5	59.5

Resistance Motor and Circuit, .616 ohms.

Impedance Motor and Circuit, .85 ohms.

$\theta = 43^\circ$.

TEST E_a.

Constant I^2R Loss.

R = .246 Entire Circuit.

Motor Terminal Voltage.	Armature Current.	K.W. Input Motor.	Motor Field.	Angle by Gears. Degrees.	Induced Motor Voltage.	Generator Output.	IZ.
		As Generators-----205					
105	50	2.2	.56	195	60	2.4	30.5
103	50	3.0	.60	195	63	3.2	30.5
103	50	3.7	.68	193	70	3.95	30.5
101	50	4.3	.75	191	74	4.55	30.5
100	50	4.8	.88	188	83	5.1	30.5
100	50	5.4	1.06	186	91	5.6	30.5
100	50	5.25	1.35	185	104	5.5	30.5
100	50	5.0	1.5	186	108	5.2	30.5
101	50	4.5	1.7	187	114	4.7	30.5
104	50	3.75	1.85	188	118	4.0	30.5
104	50	3.25	1.92	189	119	3.5	30.5
107	50	2.5	2.00	192	121	2.62	30.5

Resistance Motor and Circuit, .184 ohms.

Impedance Motor and Circuit, .608 ohms.

$\theta = 72\ 1/2^\circ$.

TEST E_b.

Constant I^2R. Loss.

R = .246. Entire Circuit.

Motor Terminal Voltage.	Armature Current	K.W. Input Motor.	Motor Field.	Angle by Gears. Degrees.	Induced Motor Voltage.	Generator Output. K.W.
		As Generators-----204				
106	40	2.2	1.9	192	119	2.4
104	40	3.0	1.75	170	115	3.2
104	40	3.5	1.6	189	111	3.7
102	40	4.0	1.45	188	107	4.2
103	40	4.35	1.34	187	103	4.5
105	40	2.0	.70	198	71	2.2
105	40	2.7	.77	196	75	2.8
103	40	3.25	.82	194	78	3.45
103	40	3.9	.95	192	86	4.1
99	40	4.25	1.0	189	88	4.3
102	40	4.25	1.15	188	96	4.5
		As Generators-----204				
106	30	2.0	.88	198	83	2.0
105	30	2.6	.95	196	86	2.65
105	30	2.95	1.2	194	98	3.00
104	30	3.35	1.25	192	100	3.40
103	30	3.2	1.35	192	103	3.25
104	30	3.0	1.42	193	106	3.1
105	30	2.9	1.54	193	109	2.95
106	30	2.5	1.59	194	111	2.6
105	30	2.4	1.62	195	112	2.45
106	30	2.25	1.66	195	113	2.25

Resistance Motor and Circuit, .184 ohms.
Impedance Motor and Circuit, .608. $\theta = 72\ 1/2^{\circ}$.

TEST E_c.

Constant I^2R. Loss.

R = .68 Entire Circuit.

Motor Terminal Voltage.	Armature Current.	K.W. Input Motor.	Motor Field.	Angle by Gears. Degrees.	Induced Motor Voltage.	Generator Output. K.W.
		As Generators-----204				
90	35	2.1	.57	199	59	2.75
85	35	2.8	.70	192	71	3.5
87	35	3.1	.90	188	83	3.75
90	35	2.8	1.15	187	95	3.5
93	35	2.6	1.35	176	103	3.25
97	35	2.15	1.52	187	109	2.80
		As Generators-----200				
84	50	2.2	.32	197.5	38	3.75
77	50	3.0	.40	190.0	46	4.6
77	50	3.35	.46	187.0	51	5.0
75	50	3.9	.70	182.0	71	5.5
		As Generators-----200				
93	50	2.3	1.95	180.0	120	4.25
93	50	2.65	1.76	182	105	4.2
90	50	3.0	1.60	183	111	4.6
85	50	3.3	1.47	183	107	4.65
82	50	3.7	1.30	179	102	5.25

Resistance Motor and Circuit, .618 ohms.

Impedance, Motor and Circuit, .846 ohms.

$\theta = 40^\circ$.

TEST E_d.

Constant I^2R Loss.

R = .436 Entire Circuit Test E_d.

Motor Terminal Voltage.	Armature Current.	K.W. Input Motor.	Motor Field.	Angle by Gears. Degrees.	Induced Motor Voltage.	Generator Output K.W.
		As Generators-----202				
100	30	2.0	.82	199	78	2.2
97	30	2.9	.96	193	86	3.2
98	30	3.15	1.2	190	98	3.4
98	30	2.95	1.28	190	101	3.2
99	30	2.7	1.35	190	103	3.0
100	30	2.6	1.42	192	106	2.75
101	30	2.4	1.50	193	108	2.6
102	30	2.2	1.55	194	109	2.4
		As Generators-----202				
96	45	2.1	.5	199	55	2.8
94	45	2.9	.55	195	59	3.5
91	45	3.8	.72	190	72	4.4
90	45	4.4	.96	185	86	5.0
90	45	4.5	1.00	184	88	5.25
92	45	4.2	1.34	185	113	4.8
95	45	3.6	1.50	186	98	4.5
98	45	3.2	1.65	187	111	3.8
100	45	2.5	1.80	188	116	3.3

Resistance Motor and Circuit, .374. ohms.

$\theta = 57^{\circ}$.